Empiricism vs Logicism

Akira Kanda

kanda@cs.toronto.edu

Peter Apostoli

apostoli@cs.toronto.edu

History of Conflict

Middle age cosmology produced demonstrably false predictions applying logical reasoning to experimentally unsubstantiated assumptions.

To resolve this problem, empiricism was developed as a protocol to extract general statements, mostly in the form of mathematical equations, from gathered experimental results. In the midst of the Religious Reformation, criticism on middle age Catholic cosmology went too far to dismiss logical reasoning as *metaphysics.*

Ironically it was the further development of physics itself which gave rise to Cantorian Set Theory which brought logic back to the centre of mathematical science. George Cantor, in his attempt to show the uniqueness of the Fourier expansion developed abstract set theory. Fourier calculus was a most powerful mathematical tool for physics and engineering in the late 19Th century.

Define a set to be *well-founded* if there is no infinite descending chain of membership relations starting from any element of the set. Let W be the set of all well-founded sets. Is W well-founded? If yes then $W \in W$. So, we have an infinite chain of membership relation

$$é\ é\ é\ . \in W \in W \in W.$$

So, we have an infinite descending chain of membership relation in W. Thus W is not well-founded. If W is not well-founded, then it contains at least one element from which an infinite chain of membership relation descend. So, it is not the set of all well-founded set. This paradox due to Miramanov establishes that Cantorian Set Theory is logically inconsistent.

As set theory was considered to be the foundation of mathematics, the discovery of inconsistency of this theory sent a shock wave through mathematics community. This gave rise to the *formalism* of David Hilbert. His idea was to formalise mathematics, foundation of mathematics in particular, so that we could prove its consistency.

To this end, Turing and Godel presented mathematical formulation of mathematical proof as computation. Ironically Godel, using his theory of computation, aka recursive function theory, proved that Hilbertôs project is untenable at the pain of contradiction.

Despite this negative result which shuttered the plan of Hilbert, the theory of computation took off as a substantial field of mathematics which in turn lead to the development of what we now call *computers*. Through computer science and metamathematics, logic established a solid place in the 20Th century science and engineering.

Parallel to this development, theoretical physics entred a new stage where traditional empiricism is not sufficient anymore.

The Hertz-Maxwell em field theory introduced a new concept which transcends empiricism. Electric field is a spatial distribution of electric force per unit charge whose direction is relative to the sign of the unit charge. The Maxwell-Hertz theory of em-waves introduced waves whose magnitude is relative to the magnitude of the receiver of the wave.

This clearly shows that at the end of the 19Th century, theoretical physics stepped into the domain of what physicists call metaphysics. This new trend continued deep into the 20Th century. For example, QM operates as a theory of self-adjoint operators over complex Hilbert space whose eigenvector analysis under the reduction of state vector hypothesis of von Neumann yields probabilistic distribution of expected values of measurement. In relativity theory, we consider multiple geometric spaces which move relative to each other and try to capture the effect of such motion in kinematics and dynamics.

Karl Popper and Inconsistency of Physical Theories

It was Karl Popper who seriously considered a fundamental difference between empirical truth and metaphysical truth. Empirical truth is contingent while metaphysical truth is absolute. Empirical truth is relative to the empirical knowledge which accumulates as the time passes. So, a general statements obtained via induction upon gathered experimental results may not be true as we gather more experimental data.

On this basis, and on the limitation on our ability to gather data, Popper correctly pointed out that all physical theories eventually come into conflict with experimental findings (aka empirical inconsistency). He proclaimed that this *falsifiability* is the nature of empirical theories. On this ground he rejects mathematics as a physical theory for example.

As a logician, Popper noticed that logically inconsistent physical theories pause a serious problem. Due to the material implication $P \rightarrow Q$ with the following truth table:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

once we prove a contradiction $A \wedge \neg A$ which is F, using modes ponense

$$\frac{A, A \rightarrow B}{B}$$

we can prove *anything*. Among logicians, this phenomenon is known as *deductive explosion*. This material implication disputed at the elementary stage of the development of mathematical logic became accepted as standard to have logical equivalence $P \leftrightarrow Q$ as $P \rightarrow Q \ \wedge Q \rightarrow P$.

This means that a logically inconsistent theory can contradict any experimental finding *A* by proving $\neg A$. So without carrying out any experiment, we already know that the theory is empirically inconsistent and thus useless for physics . Popper rightly proclaimed that all logically inconsistent theories must be excluded from physical theories.

Up until the 20Th century, theoretical physics did not encounter major problems of logical inconsistency. Appearances of the empirical inconsistency of physical theories were dealt with Hegelian dialectic. It was also agreed that they do not change theories until the empirical inconsistency becomes unbearable.

Moreover, that Popper´s falsifiability thesis also implies that there is no such thing as experimental verification of physical theories is largely unnoticed.

Axiomatic definition of SR

Special Relativity Theory assumes a basic concept of inertial frames (coordinate systems). Here two inertial frames *A* and *B* move relative to each other with speed *v* if every point of *A* moves in *B* with speed *v* and vice verse.

This concept is supported by several further assumptions.

(1) Both *A* and *B* are equipped with clocks and rods to measure their own time and distance.

(2) It is assumed that clocks of *A* and *B* and rods of *A* and *B* are identical. Without this, there is no physical connection between these two frames.

(3) The relative speed *v* is measured in each frame using these rods and clocks.

(4) At each moment, each point in *A* is also a point in *B*. Without this assumption, there is no physical connection between *A* and *B*.

It is further assumed that the relative speed between two frames is not time varying. This means that no frames accelerate relative to each other. This restriction comes from the following observation:

Suppose bikers A and B run in parallel with the same speed on a road. Assume that A throws a ball at B. Then from bikers´ perspective, the ball moves the distance between them. However, from the perspective of the road, the ball moves diagonally covering a longer distance. So, distance is not absolute. It is a *relative* physical quantity. Upon this, there is an urge to maintain laws of physics stay invariant under the choice of inertial reference frames. It can readily be seen that the second law of motion is not preserved among accelerating reference frames.

Under the concept of inertial frames, which is not as straight forward as one expects, SR is defined as an axiomatic theory with axioms {*d=vt, PR, CSL*} where *d=vt* is the kinematics equation of motion, PR is the Principle of Relativity which says that laws of physic are invariant under the choice of inertial frames, and CSL is the Constancy of Speed of Light which says that the speed of light is c in all inertial frames.

Logical Inconsistency of SR: Level One

Logical inconsistency of SR appears at many different logical levels. Most fundamental inconsistencies appear as follow:

[1] Assume that a train runs on a track with a constant speed. When the trainôs front and tail touch the track at points *F* and *R* respectively, lightning strikes *F* and *R*. An observer located in the middle of the train will observe that *F* and *R* are hit at the same time as they are point of the frame of the train. But at the same time, due to CSL, he will observe that they are hit at different time as they are points of the frame of the track moving in his frame.

[2] In the derivation of the Length Contraction by Einstein, when we consider that the point from which the light is emitted is a point of the frame of the moving mirror, we have no LC.

[3] Assume bikers A and B run in parallel with the same speed on a road. Assume A sends a light signal to B. From the perspective of the bikers, B will receive the signal. But from the perspective of the road, light will miss B.

Logical Inconsistency of SR: Level II

A full development of SR appears after the derivation of the Length Contraction and Time Dilation which leads Lorenz Transformation. At this stage many contradictions appear.

[4] Assume a train runs on a track with constant speed *v*. As Einstein pointed out, we can place synchronised clocks along the track as follows: Make a protocol that at 12.00, A light is flashed at a location say *L*. At a location *B* with *LB*=*d*, we set the clock at 12:00+ *d/c* when the light flash is observed. Now assume that operators X and Y on the train one at the front and one at the rear of the train mark the track when they see 13:00 on the clock situated on the track right in front of them. Let *C* and *D* be the marked points on the track. Then an observer on the embankment will measure the length of the train as the length of the line segment *CD*. But according to the LC, he will observe the length of the train to be $d\hat{o} = \sqrt{1-\frac{v^2}{c^2}}d$.

[5] Consider the òmuon decay problemò. We first measure the speed *v* of the muon created at the ozon layer heading towards sea level in our (seal levelôs) frame by measuring the displacement of the muon in out frame in out time interval *t* as *v* = *d/t*. According to this measurement, muon expires before it reaches the sea level covering the distance *d*. But according to the Time Dilation, we will observe that muon,s time slows down and we will observe that muon reaches the sea level before it expires. The question is what happens when muon and sea level meet? In which time muon meets the sea level? This contradiction is a logical consequence of the axioms of SR and cannot be removed by consulting with the experimental fact that muons do exist at sea level.

[6] SR is developed upon many thought experiments most of which use emission and reception of light. In further development, the *light cone interpretation* invalidates such thought experiments as it claims that there is no causality between the emission and reception of light.

Paradox [4] and [5] has a serious implication. It shows that depending upon the method of measurement, the same physical quantity in the same frame will get different values. *This pauses a fundamental question on the validity of the empiricism itself.*

Inconsistency of SR: Level III

[7] Einstein deduces the Lorenz Transformation as follows:

(1) $d=vt, PR \Rightarrow GT$ *Galilean Transformation*

(2) $d=vt, PR, CSL \Rightarrow GT$ *Monotonicity*

(3) $d=vt, PR, CSL \Rightarrow LC$ *Length Contraction*

(4) $d=vt, PR, CSL \Rightarrow TD$ *Time Dilation*

(5) $GT, LC, TD \Rightarrow LT$ *Lorenz Transformation 0*

(6) $d=vt, LC, TD \Rightarrow LT$ *Lorenz transformation 1*

But from (6) and (2), we have the following contradiction

(7) $d=vt, PR, CSL \Rightarrow LT \wedge GT.$

This level of contradiction can be reduced to a lower level issue. GT deduces Galilean addition of speed which contradicts CSL. This with (1) implies that adding CSL to {*d=vt,PR*} makes the resulting theory SR inconsistent.

The same problem manifests as a striking invalidation of the loss of simultaneity argument.

[8] Assume a train runs on a track with a constant speed. On the embankment, there are two points say *A* and *B*. When an observer on the train is on the right bisector of the line segment *AB*, lightning strikes *A* and *B*. As A and B are points of the embankment, Einstein rightly concluded that the observer will see that A and B are hit at different time. But an observer on the embankment at the mid point of AB will see that they are hit at the same time. This is how Einstein concluded the loss of simultaneity. It is unfortunate that Einstein overlooked that for the observer on the train did not use CSL. With CSL, we have a different conclusion. The observer on the train will see that A and B are hit at the same time. Therefore there is no loss of simultaneity.

Inconsistency of Relativisitc Wave Theory

Relativistic wave theory consider Lorenz transformation of mathematical waves. Considering that inertial reference frames are mathematical representations of vacuum, it is implicit that waves this theory consider are those which travel through vacuum.

[9] Through transformation of waves, this theory establishes that

$$v_g = c^2/\omega^2$$

where v_g is the group speed and ω is the phase speed of the transformed wave. This means that unless $v_g = \omega = c$ either group speed or the phase speed exceeds the speed limit *c* of SR. Even though this makes sense for the light wave with $v_g = \omega = c$, in general this contradicts SR.

De Brogli in his Ph.D. thesis dealt with this problem using a fact of classical mechanics that waves carry their energy with group speed. He thus claimed that by assuming $v_g \leq c$, we can resolve this apparent contradiction. There are two problems with his solution. First, waveôs energy in classical wave mechanics is determined relative to the mechanical nature of wave medium. Here, we are considering waves which travel through vacuum. Moreover, granted that we can abstract classical waves dropping medium to mathematical waves with group speed less than *c*. Still we have sound wave which has $v_g = \omega$ which is less than *c*.

Invalidity of Relativistic Dynamics

In his attempt to extend SR beyond kinematics to dynamics, Einstein first assumed the conservation of classical momentum and obtained that a particles with rest mass m_0 moving with speed *v* will be observed to have relativistic momentum $p = mv$ where $m = m_0 / \sqrt{1 - v^2/c^2}$.

From here he went on to derive relativistic law of motion

$$F = \frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt} \text{---------} (F).$$

According to Einstein, what makes relativistic dynamics different from the classical dynamics is this second law of motion.

Lerwill pointed out that a vector version of (*F*) will assert that the direction of force applied and that of acceleration caused need not agree. As a physicists Lerwill went on to find such examples in electro-dynamics. However, he at least gave a warning on the validity of (*F*).

Einstein went on to derive

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

from (*F*). This equation which made Einstein a household name governed entire 20Th century physics.

Inspired by Lerwill, we questioned (*F*) itself to come up with the following conclusion:

[10] In (*F*), v is a relative speed of inertial frames and it is not time variant. This reduces (*F*) to simply *F=0*. This makes the whole development of relativistic dynamics collapse. So, SR dynamics does not support $E = mc^2$.

Inconsistency of Relativistic Theory of Photons

Putting aside the above discussed issue of the invalidity of relativistic dynamics, let us consider the issue of Einsteinôs relativistic theory of photons. Relying upon the *particle-wave duality* of light, Einstein tried to introduce reintroduce light as particles into SR. A major difficulty was that this particle has speed c in all frames. This difficulty was indeed twofold:

(1) A particle which is in move in all inertial frames transcends SR´s equations which holds only for which has its own rest frame.

(2) Due to the reciprocal of the gamma factor, SR´s equation assume that the speed of the particle is less than c.

To deal with (1), Einstein just assumed that all equations of SR holds for photons. Moreover, to deal with (2), he assumed that the rest mass of a photon is zero. He thought that this would prevent the rest mass $m = m_0 / \sqrt{1 - v^2/c^2}$ of photon diverge. He further hoped that this will prevent the relativistic energy $E = m_0 c^2 / \sqrt{1 - v^2/c^2}$ of photon diverge as $E = 0/0$. Einstein thought that $0/0$ is indeterminate as so is $x \times 0 = 0$. For him all of this was a good opportunity to introduce multi-valued energy assignment of Einstein-Planck equation $E = hv$ for photons.

It was unfortunate that $0/0$ is not indeterminate. It is undefined, not a number. The proof goes as follows: Assume $0/0$ is a number. Then following the tradition of mathematics, the new number must follow the laws of old numbers. So,

$$3 \times \frac{0}{0} = \frac{3 \times 0}{0} = \frac{0}{0} = 1.$$

Also

$$3 \times \frac{0}{0} = 3 \times 1 = 3.$$

Thence, 3=1.

Adamek et. al. proposed to rewrite $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ as $m_0 = m\sqrt{1 - \frac{v^2}{c^2}}$

and $E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$ as $mc^2 = E\sqrt{1 - v^2/c^2}$ to avoid $0/0$.

[11] It is unfortunate that then the LC formula becomes $d = \sqrt{1 - v^2/c^2}\, d'$. By setting v=c, have a contradiction as both *d* and *d'* are both positive numbers. This means after we apply the remedy proposed by Adamek et al, we still have the relativistic theory of photons inconsistent.

Invalidity of General Relativity Theory

[12] Assume that *A* accelerates away from *C* with rate *a* and *B* accelerates away from *C* with rate *b* in the opposite direction. Then *B* accelerates away from *A* with rate *a+b*. We can not explicate this using force. This means that he force-acceleration duality as per the second law of motion fails. This implies that the Principle of Equivalence fails too.

Prof. Brewer cautioned us for using classical addition of acceleration. However, as relativistic dynamics is invalid, there is no relativistic addition of acceleration.

[13] Einstein failed to unify classical EM field theory and GR. This also implies that PE is invalid.

[14] By representing motion as graphs in the 4D space time, GR fails to capture the gravitational dynamics of multi-body systems such as the universe. Gravitation changes the spatial distribution of masses and GR can not deal with this. Einstein introduced cosmological constant to deal with the expansion of the universe. But expansion of the space and internal gravitational dynamics are different things.

After all, this theory, if it is valid, is good for test particles moving in a large gravitational systems, for example the double bend of light around sunôs gravitational field.

What is Light?

Modern understanding of light came from Maxwellôs theory of em field. Under the assumption that there is no conducting current, the speed of light as em wave is shown to be determined by the parameter for the em medium (ε, μ). From this Maxwell calculated that the speed of light in vacuum is c.

What is overlooked here is the empirical fact that in non-vacuum medium such as glass, where there is no conducting current, the speed of light is dependent upon the wave length of light. This indicates that there is something wrong with Maxwellôs em field theory and theory of em waves. This in turn calls in questioning the established speed of light in vacuum. The derivation of the speed of em wave in vacuum and that in non-vacuum where there is no conducting current varies only in the medium parameter (ε, μ).

Moreover, the black body radiation problem is generally taken as the victory of Quantum Mechanics and the reality that Maxwellôs theory failed empirically is unnoticed. Contrary to common belief, QM is not a solution to this shortcoming of Maxwellôs theory. Indeed, Maxwellôs theory is not a classical limit of QM.

What is Photon?

In our current understanding, there are at least three mutually contradicting concepts of photons in contemporary physics. It appears that contemporary physicists are not fully aware of this, or at least are not concerned about this.

(1) Planck quantised radiative energy as photons. This was done without any ontological substantiation. It was just a convention to get the number correct for the infamous black body radiation problem.

(2) Einstein proposed photons as quanta of light waves. Einstein failed to present the process of quantizing em waves to obtain photons. As light travels with speed c, mechanically Einstein assumed that c is the speed of light. His attempt to negotiate the difficulty of having relativistic mass and relativistic energy diverge by assuming that the rest mass is zero created a contradiction at the length contraction formula $d = \sqrt{1 - v^2/c^2}\, d'$.

(3) By Fourier expansion, Dirac quantized em fields to obtain photons. As em fields and em waves are different things of different category, contrary to common belief there is little connection between Diracôs photon and Einsteinôs photon.

An advantage of (1) is that at least it has a classical limit. By taking the quantum number large, we can link this quantization to classical energy. It appears to be that (2) is most futile. For em waves, we cannot apply the second quantization of Dirac as Fouries expansion of em waves will yield nothing material. As em wave equations are not energy equations, they are in fact em field equations, it is not possible to apply the first quantization of Von Neumann either. So, Einsteinôs quatisation of em waves is just a speculation without substantiation which contradicts SR. Unlike (1), neither (2) nor (3) seems to have classical limit. After all em waves are classical entities.

What is the Uncertainty Principle

According to Heisenberg, uncertainty principle is a mathematical representation of the impact a measurement gives to the observed system. This impact, when two measurements are not commuting, appears as the uncertainty in precision among these measurements. So, this principle applies only to two non-commuting observables. Astoundingly it is Quantum Physicists themselves who violated this principle. They consider the uncertainty applied to the pair of time and energy. They claim that this uncertainty is a prime one playing as fundamental role as the uncertainty between momentum and speed. Time is no a measurable as we cannot represent it by self-adjoint operators.

Indeed, the way this uncertainty of energy and time is introduced is completely out of quantum mechanics. This was done by considering a classical wave phenomena that there is an inherent uncertainty associated with counting the number of cress of waves. From this they derive an *uncertainty* between the frequency and time. By relating frequency to energy of light through Planck-Einstein equation, they obtain an uncertainty between energy and time. The problem with this is that the uncertainty in counting the number of cress has nothing to do with the quantum uncertainty caused by the impact a measurement gives to quantum system. By putting a *quantum* twist to this purely classical uncertainty using Planckôs quantization of energy, one cannot make what is understood as the Heisenberg uncertainty which is formulated by von Neumann.

What about Schrodingerôs Cat?

Schrodinger presented his Cat paradox to bring down the standard QM as per von Neumann. This was a serious argument to establish that standard QM lacks in classical limit and consequently it fails empirically. It was disappointing that the mainstream QM community just ignored it as an entertainment and passed it to popular science.

What is Next?

After being this much critical to the established theories of contemporary physics, it is natural to wonder what we should do to deal with these problems. Let us list some of the possibilities.

(1) It is interesting that despite fundamental failures of existing theories such as SR and GR, it appears that some of their predictions, regardless of the faulty derivation, are empirically acceptable. This is where theories like DU play an important role. Tuomo Suntola has shown that many of such predictions can be correctly deduced from DU.

(2) DU also predicts how such existing theories should fail to agree with experimental results by producing correct version and showing the match with the empirical result.

(3) It must be possible to revisit these failed theories and reformulate them correctly as a sub-theory of DU. For example GR was a wrong theory of global universe. SR went into utter confusion because it had no global theory to induce the relativistic arguments.

(4) It appeared that the attempt to unify existing theories non-vacuously (logically inconsistent theories are already vacuously unified due to the deductive explosion) failed. At least partial unification of Quantum Mechanics, em theory and relativity theory has been done under DU. Further development on this line is important.

(5) It also is important that physicists through all of this learn that pure empiricism had better days and understanding through rational metaphysical thinking is essential to avoid the current crisis happens again.