# Quantum Physics at the Crossroads of Natural Sciences, Philosophy, and Mathematics

J. Mikael Karimäki

Aalto University, Department of Mathematics and Systems Analysis

Helsinki, 21 May 2016

# Structure of Presentation

# List of Symbols

Some symbols used in this presentation:

- $h$ Planck constant
- $\hbar := h/(2\pi)$ Reduced Planck constant
- $k$ Boltzmann constant
- $G$ Gravitational constant
- $e$ Unit charge
- $c$ Speed of light in vacuum
- $\mu_0$ Permeability of vacuum
- $\epsilon_0$ Permittivity of vacuum

Modern Physics stands firmly on three solid legs:

- Quantum Physics
- Relativistic Physics
- Statistical Physics

Figure 1: Three-legged stool

Each one of these legs is tied to the concept of energy... associated with some other physical quantity, and a related constant of nature:

- $E = hf$ Quantum Physics Frequency $f$ $h$
- $E = mc^2$ Relativistic Physics Mass $m$ $c$
- $E = kT$ Statistical Physics Temperature $T$ $k$

A fourth leg on which Modern Physics stands is gravitational physics, but so far, it does not fit well with the other three (Einstein's General Theory of Relativity gives good results from mesoscopic to planetary scale, but has problems at the very small scale, and possibly at the cosmological scale):

| | | | | |
|---|---|---|---|---|
| $E = hf$ | Quantum Physics | Frequency | $f$ | $h$ |
| $E = mc^2$ | Relativistic Physics | Mass | $m$ | $c$ |
| $E = kT$ | Statistical Physics | Temperature | $T$ | $k$ |
| $E = lG^{-1}c^4$ | Gravitational Physics | Length | $l$ | $G$ |

In Relativistic Physics energy seems to obey a Pythagorean principle. Setting $c = 1$, we get:

$$E^2 = m^2 + p^2 \tag{1}$$

Question: If both rest mass $m$ and momentum $p$ appear on the same footing in this equation, does rest mass represent some kind of motion (linear, circular, or vibrational) in some extra space-like dimension?

A 4th spatial dimension has been considered, e.g., by:

- Nordström, Kaluza, Klein
- Einstein, Bergmann, Bargmann, Witten, ...
- Wesson, Randall, Sundrum, Suntola, Lehto, ...

The 4th dimension of space has been associated with:

- Electromagnetism (Nordström, Kaluza, Klein, Lehto, ...)
- Matter (Wesson, ...)
- Gravitation, gravitons (Randall, Sundrum, ...)
- An additional component of motion (Suntola, ...)

These various possibilities need not be mutually incompatible.

In many models, the 4th spatial dimension is assumed to be small and curled up (Kaluza–Klein). In some models the 4th spatial dimension is not small, but allows macroscopic motion of the 3D universe in a 4D embedding space (DU, Suntola).

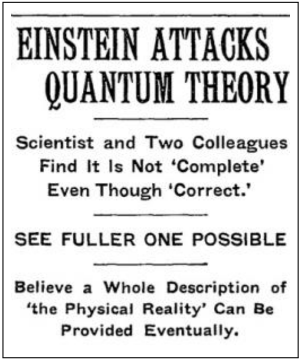
The roots of Quantum Physics lay firmly in the two other legs, Statistical Physics and Relativity:

- 1877 Boltzmann considers discrete energy levels for atoms and molecules.
- 1900 Planck introduces the quantization of energy in units of $hf$ to explain the black-body spectrum.
- 1905 Einstein explains the photoelectric effect using light quanta with energy $hf$.
- 1913 Bohr introduces his model for atoms with discrete energy levels.
- 1923 De Broglie considers quantum oscillations ($E = hf$) and concludes that relativity requires waves in space ($p = h\lambda^{-1}$).

The evolution accelerates:

- 1925 Heisenberg invents Matrix Mechanics.
- 1925 Schrödinger invents Wave Mechanics.
- 1926 Born introduces the probability rule $P = |\Psi|^2$.
- 1927 Solvay conference. The Standard or Copenhagen Interpretation of QM emerges as the generally accepted one.
- Some notable objections, e.g. by de Broglie, Einstein and Schrödinger

Figure 2: The New York Times, 4 May 1935.


EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

# Quantum Physics
## Historical Evolution

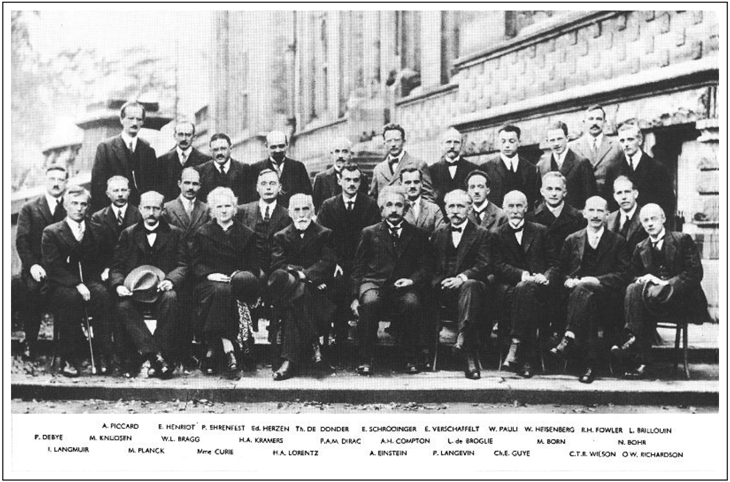



Figure 3: Participants of the 1927 Solvay Symposium.

- Progress in applications and related fields:
  - Quantum Chemistry
  - Condensed Matter Theory
  - QFT: Standard Model, String Theory, Loop QGR,
- Progress in mathematical methods:
  - Various approximative methods
  - Path Integrals, Renormalization techniques
- Discovery of many puzzling phenomena:
  - Schrödinger's cat
  - Wave-particle duality (Double-slit experiment)
  - Nonlocal correlations (EPR, Bell, Aspect, ...)
- Emergence of various interpretations, based on:
  - Ideas about quantum ontology (De Broglie, Bohm, Hiley, ...)
  - Algebraic approaches (Heisenberg, von Neumann, ...)
  - Modifying the laws of logic (Birkhoff, von Neumann, ...)

- Questions related to ontology:
  - What is the nature of the wave function?
  - What are particles, waves, quanta?
  - What is space and time?
- Nature of probability. Some possibilities:
  - Fundamental (ontological) randomness, causality broken
  - Randomness as lack of knowledge (epistemological, lack of controllability or measuralibility, “thick fingers”)
  - Randomness as illusion (superdeterministic theory that rules even over free will, “conspiration theories”)
  - Something else? (deny reality, change the rules of logic, invoke the role of consciousness, many worlds, ... what?)
- What does Quantum Physics tell us in general?
  - About other sciences? (Many of them need it, badly!)
  - About philosophical questions? (Holism)
  - About human existence? (Mind-matter problem, Free will)

# Coincidences or Connections?
## Cosmological Coincidences



Coincidences of cosmological nature:

- The sum of mass energy in the visible universe and the gravitational potential energy cancel out, causing vanishing total energy (zero-energy principle):

$$Mc^2 - GM^2/R = 0 \tag{2}$$

- The ratio of electric force and gravitational force is roughly the same as the square root of the number of charged particles in the universe:

$$\frac{e^2}{4\pi\epsilon_0 G {m_\mathrm{e}}^2} \approx \sqrt{N} \approx 10^{42} \tag{3}$$

These apparent coincidences may be true coincidences, or they may point to a Machian foundation in Cosmology, and force us to reconsider the nature of natural constants: Are they truly constant? If, not which ones are changing, which ones are not?

# Coincidences or Connections?
## Cosmological Coincidences

Let us look at the zero-energy principle in more detail:

$$Mc^2 - GM^2/R = 0.$$

Some history:

- Mentioned by Feynman as a coincidence.
- Wheeler and DeWitt: Wave function of the universe.
- Hawking writes about it in his bestseller book.
- Similar notions in Barbour's relational theories of physics.
- Applied by Suntola as the basis for the DU model.

$$Mc^2 - GM^2/R = 0.$$

Consequences of the zero-energy principle for the values of natural constants:

- It leads to: $c^2/G = M/R$.
- So, if $M/R$ changes, then either $c$, or $G$, or both, need to change also.
- Which one changes? Tuomo Suntola: $c$, Ilan Kroo: $G$.
- In any case, one needs to be very careful here!

The zero-energy principle together with Quantum Physics may imply something very strange:

- Heisenberg uncertainty principle: $\Delta E \Delta t \geq \hbar/2$.
- A quantum fluctuation has a lifetime $\Delta t \approx \hbar/\Delta E$.
- So, if $\Delta E \to 0$, then $\Delta t \to \infty$.

Edward Tryon suggested: If the universe has zero total energy, it may just have started as a quantum fluctuation from the vacuum!

# Coincidences or Connections?
## Quantum-related Constants



Let us now take a look at the Fine-structure constant:

- Dimensionless number calculated from fundamental physical constants:
$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}. \quad (4)$$
- Introduced by Arnold Sommerfeld in 1916.
- One of the most precisely known physical quantities:

$$\alpha = 7.297\,352\,566\,4(17) \times 10^{-3} \simeq \frac{1}{137}. \quad (5)$$

Tuomo Suntola's approach:

- Power radiated by a classical oscillating electric dipole:

$$P = N^2 \left(\frac{L}{\lambda}\right)^2 \frac{2}{3}(2\pi^3 e^2 \mu_0 c) f^2. \tag{6}$$

- Assume a dipole of length $\lambda$ with a single electron. The energy emitted during one cycle of oscillation:

$$E = \frac{2}{3}(2\pi^3 e^2 \mu_0 c) f \simeq \frac{2}{3} hf/1.104\,905\,316. \tag{7}$$

- Apart from the factor $2/3$, this is eerily close to the quantum mechanical formula $E = hf$.

Tuomo Suntola's approach:

We now get approximations for the Planck constant $h$ and the Fine-structure constant $\alpha$:

- The Planck constant $h$:

$$h \simeq (2\pi^3 e^2 \mu_0 c) \qquad (= h/1.104\,905\,316). \tag{8}$$

- The Fine-structure constant $\alpha$:

$$\alpha \simeq \frac{1}{4\pi^3} \qquad (= \alpha \times 1.104\,905\,316). \tag{9}$$

Tuomo Suntola's approach:

- The starting point was classical electromagnetism and antenna theory.
- Yet, we almost got the Planck constant out.

Why so close, but why not even closer? Possible explanations:

- Electron spin not included in the analysis.
- Relativistic correction needed
- Dependence on the geometry of the oscillator.
- Possible effect of extra dimensions.

# Coincidences or Connections?
## Quantum-related Constants



Ari Lehto's approach:

- Numerical analysis of physical quantities.
- Basic idea: period doubling in 3D or 4D volume.
- Value obtained for the Fine-structure constant:

$$\alpha = 2\pi \times 2^{-39/4} \simeq 7.296\,883\,5 \times 10^{-3} \tag{10}$$

- Measured value and relative error:

$$\alpha = 7.297\,352\,566\,4(17) \times 10^{-3}, \qquad \Delta\alpha/\alpha = 64 \text{ ppm}. \tag{11}$$

- We also get the Planck constant with the same accuracy:

$$h = \frac{2^{39/4}}{4\pi} e^2 \mu_0 c. \tag{12}$$

The magnetic moment of the electron (one of the most precisely known physical quantities):

- Empirical value:

$$\mu_{\mathrm{e}} = -1.001\,159\,652\,180\,91(26)\mu_{\mathrm{B}}, \tag{13}$$

$$\mu_{\mathrm{B}} = \frac{e\hbar}{2m_{\mathrm{e}}} = 9.274\,009\,994(57) \times 10^{-24}\,\mathrm{J\,T^{-1}}. \tag{14}$$

- One-loop approximation of QED:

$$\mu_{\mathrm{e}} = -\left(1 + \frac{\alpha}{2\pi}\right)\mu_{\mathrm{B}} = -1.001\,161\,409\,7\mu_{\mathrm{B}}. \tag{15}$$

- Ari Lehto's model:

$$\mu_{\mathrm{e}} = -\left(1 + 2^{-39/4}\right)\mu_{\mathrm{B}} = -1.001\,161\,335\,1\mu_{\mathrm{B}}. \tag{16}$$

Let us define the Planck mass, energy, time, and length:

$$m_{\mathrm{Pl}} := \sqrt{\frac{hc}{G}} = 5.455\,601\,7 \times 10^{-8}\,\mathrm{kg}, \tag{17}$$

$$E_{\mathrm{Pl}} := \sqrt{\frac{hc^5}{G}} = 4.903\,250\,3 \times 10^{9}\,\mathrm{J}, \tag{18}$$

$$t_{\mathrm{Pl}} := \sqrt{\frac{Gh}{c^5}} = 1.351\,362\,8 \times 10^{-43}\,\mathrm{s}, \tag{19}$$

$$l_{\mathrm{Pl}} := \sqrt{\frac{Gh}{c^3}} = 4.051\,283\,7 \times 10^{-35}\,\mathrm{m}. \tag{20}$$

(These differ from the usual definition of Planck units by a factor of $\sqrt{2\pi}$, because $h$ is used here instead of $\hbar$. However, $h$ really seems to be more fundamental than $\hbar$.)

We continue along Lehto's line of thinking:

The mass of an electron-positron pair:

$$m_{\mathrm{ee}} = 2m_{\mathrm{e}} = 1.821\,876\,7 \times 10^{-30}\,\mathrm{kg}. \tag{21}$$

Its ratio to the Planck mass:

$$\frac{m_{\mathrm{ee}}}{m_{\mathrm{Pl}}} = 3.339\,460\,6 \times 10^{-23} = 2^{-223.99419/3} \simeq 2^{-224/3}. \tag{22}$$

Relative error: $0.13\%$.

The hydrogen $21\,\mathrm{cm}$ spectral line (hyperfine transition):

$$l_{21} = 21.106\,114\,054\,13\,\mathrm{cm}, \tag{23}$$

$$m_{21} = 1.047\,193\,7 \times 10^{-41}\,\mathrm{kg}. \tag{24}$$

The ratio of its mass to the electron-positron pair mass:

$$\frac{m_{21}}{m_{\mathrm{ee}}} = 5.747\,884\,6 \times 10^{-12} = 2^{-112.02032/3} \simeq 2^{-112/3}, \tag{25}$$

and to the Planck mass:

$$\frac{m_{21}}{m_{\mathrm{Pl}}} = 2^{-112.00484} \simeq 2^{-112}, \tag{26}$$

Relative error: $0.33\%$.

To summarize:

$$m_{21}/m_{\rm ee} = 2^{-112.02032/3} \simeq 2^{-112/3}, \tag{27}$$

$$m_{\rm ee}/m_{\rm Pl} = 2^{-223.99419/3} \simeq 2^{-224/3}, \tag{28}$$

$$m_{21}/m_{\rm Pl} = 2^{-336.01451/3} \simeq 2^{-336/3}. \tag{29}$$

All of these are nearly of the form $2^{N/3}$, where $N$ is an integer.

Furthermore:

$$m_{\rm Pl} \simeq \left(\frac{m_{\rm ee}}{m_{21}}\right)^2 m_{\rm ee}. \tag{30}$$

So, we have several apparent coincidences for these mass ratios. What does this mean? Are these coincidences connected?

Let us define a Planck volume:

$$V_{\rm Pl} = (l_{\rm Pl})^3 = \left(\frac{Gh}{c^3}\right)^{3/2} = 6.649\,331\,5 \times 10^{-104}\,\mathrm{m}^3. \qquad (31)$$

We can also compute volumes for $m_{\rm ee}$ and $m_{21}$ using the Compton wavelengths $\lambda_{\rm ee}$ and $\lambda_{21}$:

$$V_{\rm ee} = (\lambda_{\rm ee})^3 = 1.785\,455\,4 \times 10^{-36}\,\mathrm{m}^3 \simeq 2^{224} V_{\rm Pl}, \qquad (32)$$

$$V_{21} = (\lambda_{21})^3 = 9.402\,099\,5 \times 10^{-3}\,\mathrm{m}^3 \simeq 2^{336} V_{\rm Pl}. \qquad (33)$$

In Ari Lehto's model this would indicate 224 doublings of $V_{\rm Pl}$ to get the electron-positron pair, and 336 doublings for the $21\,\mathrm{cm}$ spectral line.

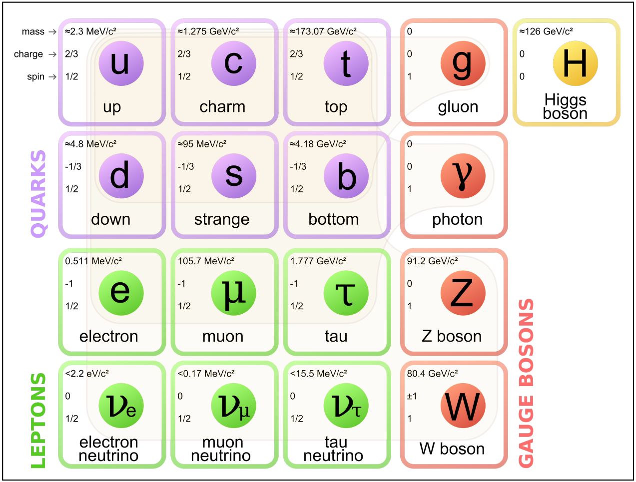



Figure 4: Particles in the Standard Model.

The masses of fundamental particles look quite arbitrary. For the charged leptons $(\mathrm{e}, \mu, \tau)$, we have:

$$m_\mathrm{e} = 9.109\,383\,56(11) \times 10^{-31}\,\mathrm{kg} \simeq 0.5110\,\mathrm{MeV}, \tag{34}$$

$$m_\mu = 1.883\,531\,594(48) \times 10^{-28}\,\mathrm{kg} \simeq 105.66\,\mathrm{MeV}, \tag{35}$$

$$m_\tau = 3.167\,47(29) \times 10^{-27}\,\mathrm{kg} \simeq 1776.8\,\mathrm{MeV}. \tag{36}$$

However, there is a strange formula by Yoshio Koide:

$$K(\mathrm{e}, \mu, \tau) = \frac{m_\mathrm{e} + m_\mu + m_\tau}{(\sqrt{m_\mathrm{e}} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}. \tag{37}$$

It is not accepted by mainstream particle physics... But it is very accurate:

- Empirical value: $K(\mathrm{e}, \mu, \tau) \simeq 6.666\,605 \times 10^{-1}$.
- Relative error: $9.4\ \mathrm{ppm}$

# Conclusions and Discussion



- Quantum Physics is founded on previous theories of Statistical Physics and Relativity.
- Many seemingly different approaches to Quantum Physics have proven useful.
- Relativistic theories have been successful from atomic to planetary scale.
- Cosmological observations and unexplained coincidences suggest that a Machian theory may be needed.
- Some natural constants may in fact be changing over time. Which ones? ($c, G, h, e, \ldots$ ?)
- Sometimes playing with numerical values may point to underlying laws and regularities. (Mendeleev!)
- Seemingly different approaches are not always incompatible.
- What is good science? How much flexibility, diversity, mutual respect and tolerance should there be? Pros and Cons?

THE END

Thank you for your attention!

# Internet Sources
## Organizations

- PDG — Particle Data Group
- NIST — National Institute of Standards and Technology
- CODATA — Committee on Data for Science and Technology
- BIPM — Bureau International des Poids et Mesures